CURVATURE OF OIL WELL INDICATOR CHARTS IN FRACTURED POROUS RESERVOIRS

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In the course of hydrodynamical investigations of absorbing strata designed to observe the drop in the level in oil wells [1], it was discovered that $Q-\Delta p$ indicator charts (here Q is the instantaneous liquid flow rate, Δp is the instantaneous excess pressure) show a characteristic curved shape (Fig. 1) recurring in general traits in most of the wells investigated. In the range of low liquid flow rates, the indicator chart appears convex facing the flow rate axis over a small chart interval, while in the range of large flow rates it is close to a straight line cutting off a terminal segment on the pressure drop axis.

The absorbing strata are fractured and vesicular sandstones characterized by very high hydraulic conductivity and rapid stabilization of conditions (in a matter of minutes). This last fact provides the basis for treating the indicator charts obtained under transient conditions as stationary flow characteristics of the absorbing stratum.

Within the framework of this assumption, the observed shape of the indicator charts can be accounted for by assuming that the initial (limiting) pressure gradient which has to be overcome before the liquid can begin flowing is detectable in porous blocks of low permeability when the stratal fluid seeps through. The reason for that could be the presence of some, even a low, limiting shear stress in the liquid, and its manifestation as a result of the interaction between the liquid and the porous medium (water) or as a result of the manifestation of capillary effects in the porous blocks (see [2-4] for more detail). Below, we present a description of the phenomenon within the framework of the overall filtration scheme in media of double porosity, as proposed in [5, 6].

The phenomenon under discussion has a lot in common with the increase in the effective thickness of the stratum when the production rate of an oil well is stepped up. It is most often related to the appearance of fracturing in a stratum leading to new partings. But the increase in effective thickness is observed not only in injection wells, but also in producing wells. It is therefore natural to assume that the effective thickness depends not on the pressure (or at least not solely on the pressure) but on the pressure gradient per se. Compelling arguments in support of that assumption are marshalled in [7]. The change in effective thickness (new partings) can be accounted for in turn by the nonlinearity of the filtration law (by the presence of an initial pressure gradient varying from one parting to the next) [8]. The special feature of what we term the fractured porous medium is that the effect of inertia-related (quadratic) terms in the law of hydraulic resistance can become substantial in flow through the cracks.

1. We consider a fractured porous medium consisting of porous slugs or blocks separated from each other over almost the entire surface by fissures. We shall assume that the porosity m of the blocks is much greater than the total volume of the fissures per unit volume of the medium, and that the hydraulic conductivity of the blocks is commensurate with the hydraulic conductivity of the system of fissures.

We now state the generalized Darcy law (law of filtration with limiting gradient) for the flow of liquid through porous blocks

$$\mathbf{u}_2 = -\frac{k_2}{\mu}\left(\operatorname{grad} p - \gamma \frac{\operatorname{grad} p}{|\operatorname{grad} p|}\right), \quad |\operatorname{grad} p| > \gamma \qquad (1.1)$$
$$\mathbf{u}_2 = 0, \quad |\operatorname{grad} p| \leqslant \gamma$$

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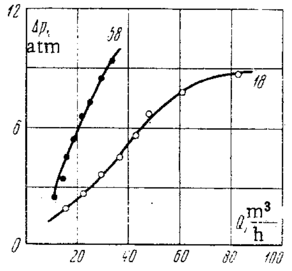

Fig. 1

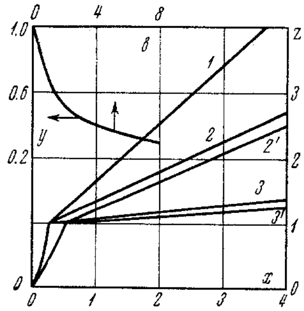

Fig. 2

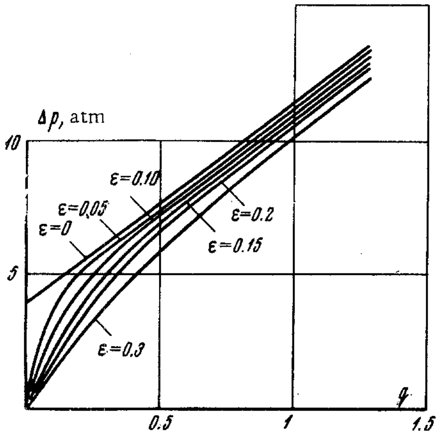

Fig. 3

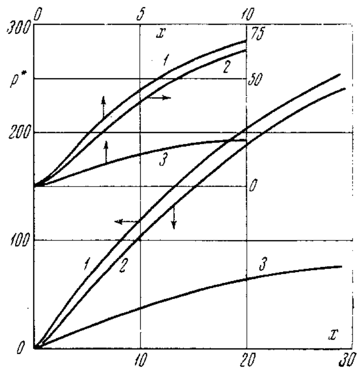

Fig. 4

Here k_2 is the permeability of the porous blocks, γ is the limiting pressure gradient for blocks, and p is the pressure in the fissures.

The existence of the limiting pressure gradient can be left out of account when treating flow through the fissures, since the opening of the fissures is much greater than the mean pore size in the blocks, but the quadratic term in the resistance law must be taken into account. We then have [9]:

$$\operatorname{grad} p = -\mu\,(1 + \beta u_1)\,\mathbf{u}_1 / k \tag{1.2}$$

Here $\mathbf{u}_1$ is the filtration rate through the system of fissures, k_1 is the fracture permeability, and β is a coefficient.

We now introduce the vector of the total liquid flow through unit area of the stratum

$$\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2 \tag{1.3}$$

Upon eliminating $\mathbf{u}_1$ and $\mathbf{u}_2$ from Eqs. (1.1)-(1.3), we obtain the relationship between $\mathbf{u}_1$ and grad p in the form

$$\operatorname{grad} p = -(\mu / k_1)(1 + \beta u)\,\mathbf{u} \qquad (u \leqslant u_0)$$

$$\operatorname{grad} p = -\left[\gamma + \frac{\mu u}{k_2} + \frac{\mu}{2\beta k_2} + \frac{\mu k_1}{2\beta k_2^2} - \frac{\mu}{2k_2\beta}\left(1 + \frac{k_1^2}{k_2^2} + 2\frac{k_1}{k_2} + \frac{4\gamma\beta k_1}{\mu} + \frac{4k_1\beta u}{k_2}\right)^{1/2}\right]\frac{\mathbf{u}}{u} \qquad (u \geqslant u_0) \tag{1.4}$$

where

$$u_0 = {}^1/_2\,\beta^{-1}\left(\sqrt{1 + 4\beta\gamma k_1 / \mu} - 1\right) \tag{1.5}$$

The total flow satisfies the continuity equation

$$\operatorname{div} \mathbf{u} = 0 \tag{1.6}$$

Equations (1.4) and (1.6) show that the flow in question is equivalent to flow through a homogeneous stratum in the case of a nonlinear filtration law of the form (1.4). Specifically, when $\beta = 0$ Eqs. (1.4) convert to the piecewise-linear filtration law

$$\mathbf{u} = -\frac{k_1}{\mu}\operatorname{grad} p, \qquad u \leqslant \lambda = \frac{k_1\gamma}{\mu}$$
$$\mathbf{u} = -\frac{k_1+k_2}{\mu}\left[\operatorname{grad} p - \gamma\frac{k_2}{k_1+k_2}\frac{\operatorname{grad} p}{|\operatorname{grad} p|}\right], \quad u \geqslant \lambda \tag{1.7}$$

On the whole, the pattern of flow has a lot in common with the flow of a viscoplastic liquid, discussed earlier [8], through a layered stratum consisting of partings of contrasting permeability. Figure 2 shows, plotted in dimensionless coordinates, $z = |\operatorname{grad} p|/\gamma$, $x = u/\lambda$, the form of the arbitrary filtration law (1.4) when different relationships obtain between the parameters $C = k_2/k_1$ and $B = \beta\lambda$, as well as the dependence $y = u_0/\lambda$ on the parameter B. Here curves 1 refer to C = 1, and curves 2 to C = 2, curves 3 to C = 10. Unprimed numbers correspond to B = 10, primed numbers to B = 2.

2. By using Eqs. (1.4) we can readily arrive at the relationship between the flow rate of liquid absorbed by the stratum and the pressure in the well. This relationship is described by the equations

$$\Delta p = \frac{\mu Q}{2\pi k_2 h} C\left[\ln\frac{1}{\rho} + Bx(1-\rho)\right], \qquad x \leqslant 1$$
$$\Delta p = \frac{\mu Q}{2\pi k_2 h}\left\{\left[C(1+B) + \frac{1+C}{2BC}\right]\left(1-\frac{1}{x}\right) + \ln x\right. \tag{2.1}$$
$$\left. - \frac{xg(1,1) - g(x,1)}{2BCx} - \frac{2}{\Delta}\ln\left[\sqrt{x}\,\frac{\Delta + g(1,1)}{\Delta + g(x,1)}\right] + C\ln\frac{1}{\rho x} + BC(1-\rho x)\right\} \quad (1 \leqslant x \leqslant 1/\rho)$$
$$\Delta p = \frac{\mu Q}{2\pi k_2 h}\left\{\left[C(1+B) + \frac{1+C}{2BC}\right]\frac{1-\rho}{\rho x} + \ln\frac{1}{\rho} - \frac{g(x,\rho)}{2BC\rho x} + \frac{g(x,1)}{2BCx} - \frac{2}{\Delta}\ln\left[\frac{1}{\sqrt{\rho}}\,\frac{\Delta + g(x,\rho)}{\Delta + g(x,1)}\right]\right\} \qquad \left(\frac{1}{\rho} \leqslant x < \infty\right)$$

Here

$$x = \frac{Q}{2\pi r_c u_0 h}, \quad \rho = \frac{r_c}{r_k}, \quad B = \beta u_0, \quad C = \frac{k_2}{k_1}, \quad \Delta^2 = (1+C)^2 + 4BC^2(1+B)$$
$$g(x,\rho) = \sqrt{\Delta^2 + 4\rho BCx}$$

Specifically, when $\beta = 0$, these equations become simplified, converting to

$$\Delta p = -Q^* C\ln\rho, \quad x \leqslant 1 \qquad (Q^* = \mu Q/2\pi k_2 h)$$
$$\Delta p = Q^* C\,[C(1+C)^{-1}(1 - {}^1/_x - \ln x) - \ln\rho], \qquad 1 \leqslant x \leqslant 1/\rho \tag{2.2}$$
$$\Delta p = Q^* C\,[C(1+C)^{-1}(1-\rho)/\rho x - (1+C)^{-1}\ln\rho], \quad 1/\rho \leqslant x < \infty$$

The characteristic shape of the indicator charts described by Eqs. (2.2) is shown in Fig. 3 as plotted for the case $r_k = 200$ m, $r_c = 0.1$ m, $\gamma = 0.02$ atm/m, $\varepsilon = 1/(1+C) = 0 + 0.3$, and q denotes the quantity $\mu Q/(2\pi h(k_1+k_2))$. One common feature of all the indicator charts is their convexity facing the depression axis and the presence of an asymptotic linear interval determined by the third equation of Eqs. (2.2) at fairly high production rates. The slope of that interval is usually determined by the hydraulic conductivity of the stratum, while the segment cut off by that interval on the depression axis is equal to $\gamma r_k C/(1+C)$, i.e., the limiting pressure gradient for the blocks can be estimated from it.

The indicator curves, while closely similar to those observed in the range of high flow rates, differ from them at low production rates in lacking a convex sector facing the flow axis. That might be due to the omission of the quadratic term in the arbitrary filtration law, i.e., in the conversion from Eq. (2.1) to the simplified Eqs. (2.2). Actually, as the indicator curves plotted in Fig. 4 show, there exists a range of values of the parameters over which the shape of the predicted indicator curves is close to the empirical shape; here $p^* = (2\pi k_2 h/\mu)\,\Delta p$. The top of the figure belongs to $\rho = 0.01$, the bottom to $\rho = 0.03$. For curves 1, B = C = 2, for curves 2, B = 10 and C = 0.5, and for curves 3, B = 10 and C = 2.

3. In the general case ($B \neq 0$), the formulas obtained are quite complicated, but still lend themselves to straightforward analytic investigation. But the analytic investigation must be conducted while entertaining additional conditions for the distinct intervals of changes in rates of flow. We designate

$$\frac{2\pi k_2 h}{\mu Q}\Delta p = P(x) \tag{3.1}$$

so that P (x) is proportional to the slope of the segment joining the instantaneous point on the indicator chart to the origin of coordinates. We find from Eqs. (2.1), to begin with, that P (x) is a continuous function, and that

$$P(0) = C \ln (1/\rho), \quad P(1) = C[\ln(1/\rho) + B(1-\rho)] \tag{3.2}$$

Consequently, P (x) is increasing on some interval $0 \leq x \leq x_m$, i.e., the indicator chart is convex facing the flow axis. On the other hand, we realize from the third formula in (2.1) that, in the limit as $x \to \infty$, we have $P(x) \to \ln(1/\rho)$. That means that when $C > (1 - B \ln \rho)^{-1}$, P (x) must necessarily attain a maximum at some finite $x_m < \infty$, i.e., there is a point on the indicator curve at which the tangent to the indicator curve passes through the origin of coordinates. We now consider values of x such that $z = \rho x$ is of the order of several units, $1 \leq \rho x \leq 10$. Then with the condition $\rho \ll 1$, we can simplify Eqs. (2.2). Assuming the condition $1 < 4BCz \ll \Delta^2$ met, we can obtain, accurate to infinitesimals,

$$\Delta p = \frac{\mu Q}{2\pi k_2 h} \ln \frac{1}{\rho} + \frac{\mu u_0 r_k}{k_2} C(1+B)\left(1 - \frac{2}{1+C+\Delta}\right) \tag{3.3}$$

This equation corresponds to a straight line cutting off the interval

$$\Delta p_0 = \frac{\mu r_k u_0}{k_2} C(1+B)\left(1 - \frac{2}{1+C+\Delta}\right) \tag{3.4}$$

on the pressure axis.

According to (1.5),

$$\gamma = (\mu u_0 / k_1)(1 + \beta u_0) = (\mu u_0 / k_1)(1 + B) \tag{3.5}$$

Hence, from Eqs. (3.4) and (3.5),

$$\Delta p_0 = r_k \gamma \left(1 - \frac{2}{1+C+\Delta}\right) \tag{3.6}$$

That means that the interval cut off on the pressure axis will enable us to estimate γ, if the value of r_k is known (or has been estimated). Finally, for the value of x_m at which P (x) attains a maximum, we have, in a first approximation, and recalling our previous assumptions:

$$x_m = \frac{C(1+B)}{C-1}\left(1 - \frac{2}{1+C+\Delta}\right) \tag{3.7}$$

The corresponding flow rate

$$Q_m = 2\pi r_c h x_m u_0 = \frac{2\pi r_c h k_2 \gamma}{\mu (C-1)}\left(1 - \frac{2}{1+C+\Delta}\right) \tag{3.8}$$

Consequently, there is qualitative agreement between the experimentally observed and predicted indicator charts. Verification of the quantitative relationship would be out of the question on the basis of the material available. Nor can we assume with complete confidence that the mechanism described provides the only possible explanation for the shape of the indicator charts observed.

4. Up to this point we have been considering a fractured porous stratum in which the pressure in the blocks and in the fissures is the same in each cross section. Another limiting model is obtained when we assume that the porous portion (the blocks) and the channels (pockets or fissures) are opened up only by the well being driven, share a common external boundary, but do not interact in the stratum cross sections. For the rate of flow through the channels, we have

$$\Delta p = LQ_1 + MQ_1^2 = \frac{\mu Q_1}{2\pi k_1 h} \ln \frac{r_k}{r_c} + \frac{\beta Q_1^2}{4\pi^2 h^2}\left(\frac{1}{r_c} - \frac{1}{r_k}\right) \tag{4.1}$$

and for the rate of flow through the porous part we have

$$\Delta p = \frac{\mu Q_2}{2\pi k_2 h} \ln \frac{r_k}{r_c} + \gamma r_k \tag{4.2}$$

The total flow rate

$$Q = Q_1 + Q_2 \tag{4.3}$$

It is readily seen that these equations agree, except for differences in notation, with Eqs. (1.2)-(1.4). We introduce the notation

$$Q_0 = {}^1/_2 M^{-1} L\left(\sqrt{1 + 4\gamma r_k M / L^2} - 1\right) \tag{4.4}$$

whereupon we get

$$\begin{gathered}\Delta p = LQ + MQ^2 \qquad (Q \leqslant Q_0) \\ \Delta p = \gamma r_k \left(1 + ax + \frac{a}{2b} + \frac{a^2}{2b} - \frac{a}{2b}\sqrt{(1+a)^2 + 4b + 4abx}\right) \qquad (Q \geqslant Q_0)\end{gathered} \tag{4.5}$$

where

$$\begin{gathered}a = \frac{k_1}{k_2} = \frac{1}{C}, \quad b = \frac{MQ^*}{L}, \quad x = \frac{Q}{Q^*} \\ Q^* = \frac{\gamma r_k}{L} = \frac{2\pi k_1 h \gamma r_k}{\mu \ln (r_k / r_c)}\end{gathered} \tag{4.6}$$

The corresponding indicator charts are represented by the curves plotted in Fig. 2.

It must be pointed out, however, that the model with the independent porous stratum is more or less at variance with the assumed rapid stabilization of the nonstationary process.

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